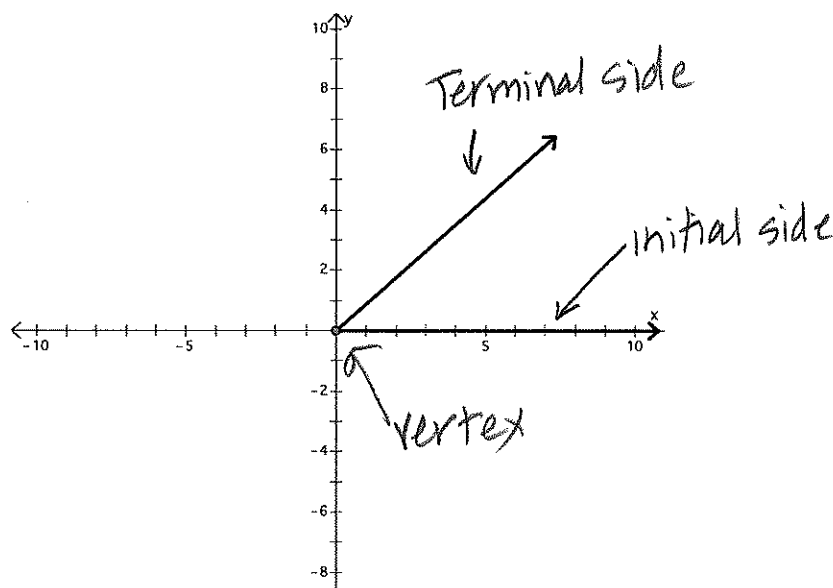


Radian & Degree Measure

Angles

- An angle is determined by rotating a ray about its endpoint.
- The starting position of the ray is the INITIAL side of the angle, and the position after the rotation is the TERMINAL side.
- The endpoint of the ray is the VERTEX of the angle.

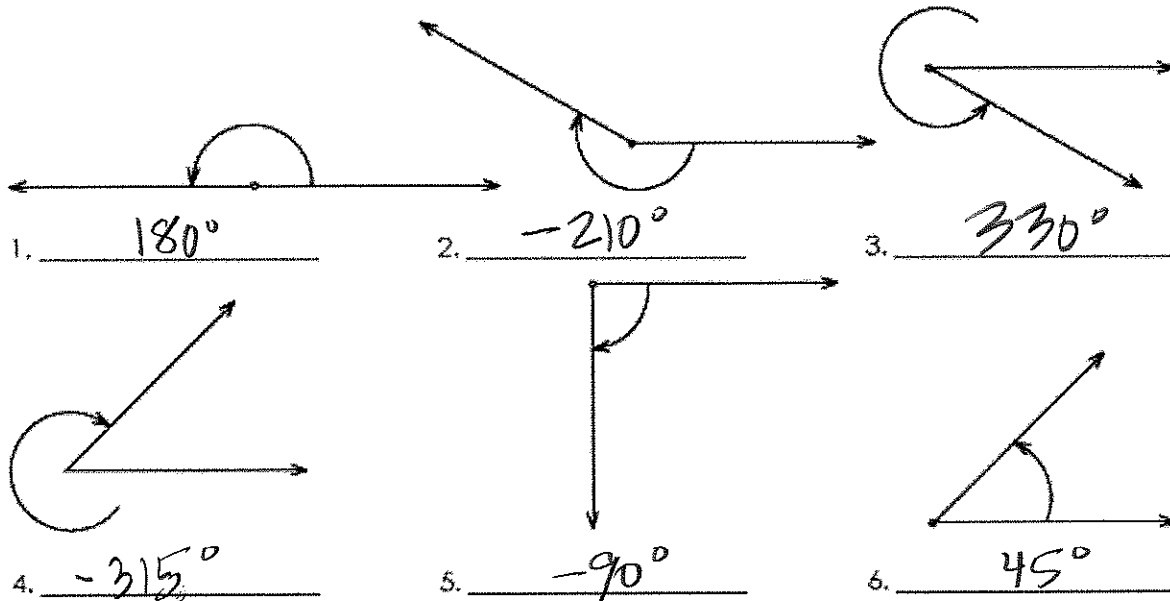
Ex 1 Label the angle with its initial side, terminal side, and vertex.



Standard Position, Positive Angles, Negative Angles

- In the coordinate plane, when the vertex of an angle is at the origin, and the initial side is on the positive x-axis, the angle is said to be in STANDARD POSITION.
- Positive angles are formed by a COUNTER- clockwise rotation, and negative angles are formed by a clockwise rotation.
- Angles are typically labeled by Greek letters such as α (alpha) β (beta) or θ (theta) as well as uppercase letters such as A, B, or C.

Ex 2 Estimate the degree measure of each of the following angles.



Ex 3 What quadrant do the following angles' terminal side lie in?

A) 130°

II

B) 350°

IV

C) -100°

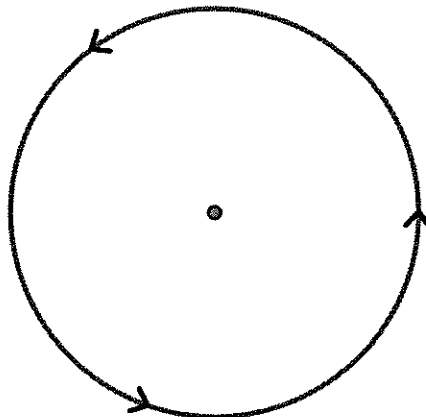
III

D) 200°

III

Radian Measure

The measure of an angle is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in **radians**. This type of measure is especially useful in calculus and is helpful in connecting linear units with angle measure.



Remember: One full revolution (360°) has measure 2π radians.

Radians compared to the degree measure

The first common way you learned to measure angles is in **degrees**, where a full (counterclockwise) revolution corresponds to 360° , a half-revolution to 180° , a quarter-revolution to 90° , and so on.

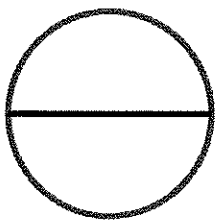
Ex 4 Given that one full revolution measures 2π radians,

$$\frac{1}{2} \text{ revolution} = \underline{\pi} \text{ radians} = \underline{180} \text{ degrees}$$

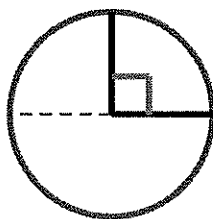
$$\frac{1}{4} \text{ revolution} = \underline{\pi/2} \text{ radians} = \underline{90} \text{ degrees}$$

$$\frac{1}{6} \text{ revolution} = \underline{\pi/3} \text{ radians} = \underline{60^\circ} \text{ degrees}$$

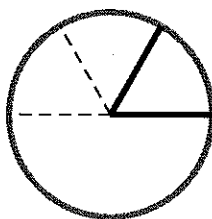
Ex 5 How many radians are shown in each diagram?



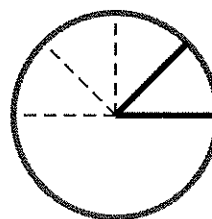
$$\underline{\pi}$$



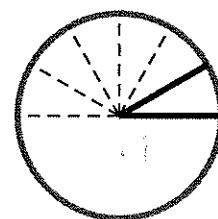
$$\underline{\pi/2}$$



$$\underline{\pi/3}$$



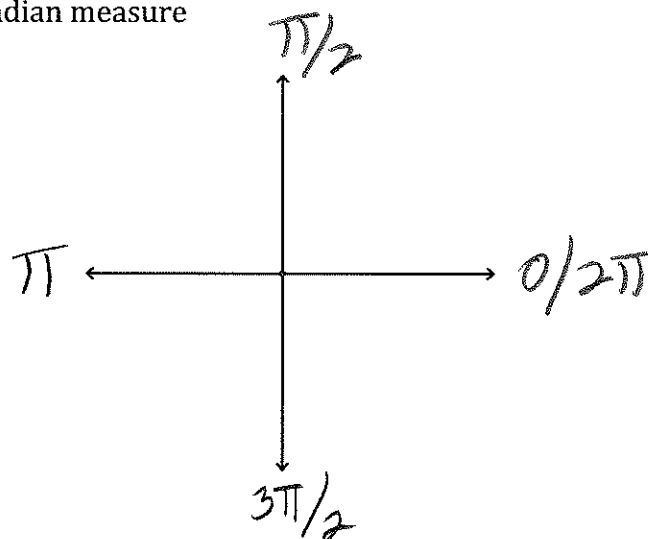
$$\underline{\pi/4}$$



$$\underline{\pi/6}$$

Ex 6 Label each quadrantal angle in radian measure

an $\angle$ whose terminal side is on an axis



Given that 2π radians corresponds to one complete revolution, degrees and radians are related by the equations:

$$360^\circ = 2\pi \text{ radians and } 180^\circ = \pi \text{ radians}$$

Therefore, $1^\circ = \frac{\pi}{180}$ radians and $1 \text{ radian} = \frac{180}{\pi}$ degrees.

Converting Degrees $\leftrightarrow$ Radians

- To convert from Degrees to Radians, multiply degrees by $\frac{\pi}{180}$.
- To convert from Radians to Degrees, multiply radians by $\frac{180}{\pi}$.

Ex 7 Most frequently used conversions:

$$30^\circ = \frac{\pi}{6} \text{ radians}$$

$$45^\circ = \frac{\pi}{4} \text{ radians}$$

$$60^\circ = \frac{\pi}{3} \text{ radians}$$

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

$$180^\circ = \pi \text{ radians}$$

$$270^\circ = \frac{3\pi}{2} \text{ radians}$$

$$360^\circ = 2\pi \text{ radians}$$

Ex 8 Convert from Degrees $\rightarrow$ Radians

$$\text{a. } 135^\circ \left(\frac{\pi}{180} \right) = \frac{3\pi}{4}$$

$$\text{b. } -210^\circ \frac{\pi}{180} = -\frac{7\pi}{6}$$

Ex 89 Convert from Radians $\rightarrow$ Degrees

$$\text{a. } -\frac{5\pi}{6} \text{ radians}$$

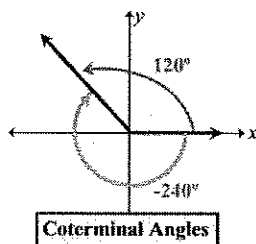
$$-\frac{5\pi}{6} \cdot \frac{180}{\pi} = -150^\circ$$

$$\text{b. } \frac{2\pi}{3} \text{ radians}$$

$$\frac{2\pi}{3} \left(\frac{180}{\pi} \right) = 120^\circ$$

Coterminal Angles

When two or more angles have the same initial and terminal sides, they are “coterminal”



In radians, you can find an angle that is coterminal to a given angle θ by **adding or subtracting** 2π (1 full revolution) or any **multiple of** 2π

Ex 4 Find 3 angles coterminal to 110°

$$110^\circ + 360^\circ = 470^\circ$$

$$+ 360^\circ = 830^\circ$$

$$110^\circ - 360^\circ = -250^\circ$$

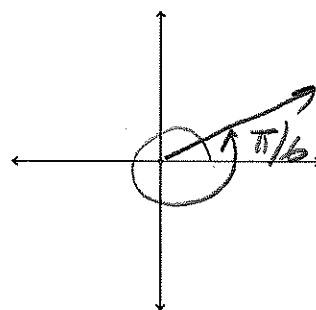
How many angles are coterminal to 110° ? ∞

Ex 5 Finding Coterminal Angles in radian measure

- a. For the positive angle $\frac{13\pi}{6}$, find a positive coterminal angle, and sketch.

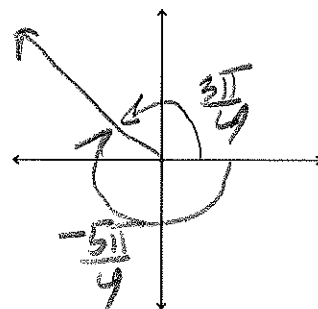
$$\frac{13\pi}{6} - 2\pi$$

$$= \frac{13\pi}{6} - \frac{12\pi}{6} = \frac{\pi}{6}$$



- b. For the positive angle $\frac{3\pi}{4}$, find a negative coterminal angle, and sketch.

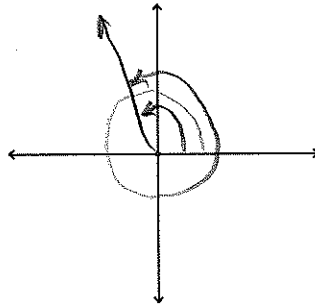
$$\frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{5\pi}{4}$$



Section 4.1 HW Day 1 : p.290-2 #7, 13, 17, 19, 25-37 odd, 51-69 odd

Additional Practice

- 1) Find a positive coterminal angle for $\theta = 100^\circ$. Sketch both angles in standard position.

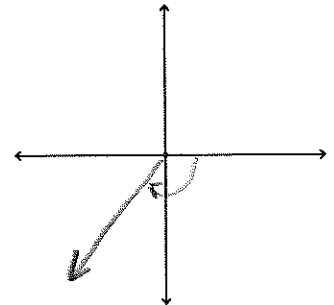


$$100 + 360 = 460^\circ$$

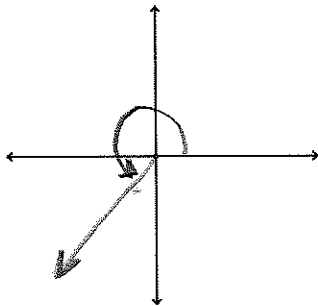
- 2) Sketch -120° in standard position. Find two more negative angles that are coterminal with -120° .

$$-120 - 360 = -480^\circ$$

$$-480 - 360 = -840^\circ$$



- 3) For the negative angle $-\frac{2\pi}{3}$, find a positive coterminal angle, and sketch.



$$\frac{4\pi}{3}$$

$$-\frac{2\pi}{3} + 2\pi$$

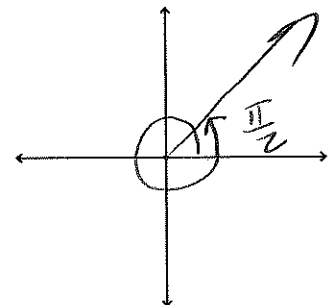
$$-\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{4\pi}{3}$$

- 4) Determine the quadrant in which $\frac{9\pi}{8}$ lies.

III

$$\frac{9\pi}{8} \approx \pi$$

- 5) Sketch $\frac{5\pi}{2}$ in standard position.



- 6) Convert from degrees to radians: 150°

$$150 \left(\frac{\pi}{180} \right) = \frac{5\pi}{6}$$

- 7) Convert from radians to degrees: $\frac{5\pi}{4}$

$$\left(\frac{180}{\pi} \right) = 225^\circ$$

More from Lesson 4.1 with Applications!

Degrees, Minutes, Seconds vs Decimal Degrees

$$\text{Minute} = 1' = \frac{1}{60}^{\circ}$$

$$\text{Second} = 1'' = \frac{1}{60}' = \frac{1}{3600}^{\circ}$$

Ex 1 Convert from DMS to Decimal Degrees: $74^{\circ} 8' 14''$

$$74 + \frac{8}{60} + \frac{14}{3600} = 74.1372$$

Ex 2 Convert from Decimal Degrees to DMS: 34.817°

$$\begin{aligned} &= 34^{\circ} (.817) \times 60' \\ &= 34^{\circ} 49.02' \\ &= 34^{\circ} 49' (.02) \times 60 = 34^{\circ} 49' 1.2'' \end{aligned}$$

Ex 3 You try:

a. $34^{\circ} 51' 35'' \rightarrow$ decimal degrees

$$34.85972^{\circ}$$

b. $102.3771 \rightarrow$ DMS

$$102^{\circ} 22' 37.56''$$

Arc Length

For a circle of radius r , a central angle of θ intercepts an **arc of length s** given by

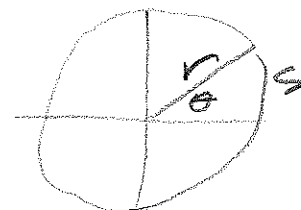
$$s = r\theta \quad \text{where } \theta \text{ is measured in RADIANS}$$

Ex 6 A circle has a radius of 5 cm.

Find the length of the arc intercepted by a central angle of 150° .

convert to radians: $150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

$$s = 5\left(\frac{5\pi}{6}\right) = \frac{25\pi}{6} \text{ cm}$$



Linear and Angular Speeds

Consider a particle moving at constant speed along a circular arc of radius r . If s is the length of the arc traveled in time t , then the **linear speed** v of the particle is

$$v = \frac{s}{t} = \frac{\text{arc length}}{\text{time}} \quad \frac{\text{ft, m, mi, etc.}}{\text{sec, min, hr, etc.}}$$

Also, if θ is the angle (again, in radian measure) corresponding to the arc length s , then the **angular speed** ω of the particle is

$$\omega = \frac{\theta}{t} = \frac{\text{central angle}}{\text{time}} \quad \frac{\text{radians}}{\text{sec, min, hr, etc.}}$$

Ex 7: Ms. Chen and her nephew, Brady, went on a Ferris wheel for the first time this summer! It was designed for 4 year olds, so it had an 80 foot diameter and made about 1.25 revolutions per minute. Find the angular and linear speeds of the wheel so Ms. Chen can tell Brady how FAST they went!

$$v = \frac{s}{t} \quad s = r\theta$$

$$\theta = 2\pi(1.25) = 2.5\pi$$

↑
each revolution

$$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{40(2.5\pi)}{1} = 100\pi \text{ ft/min}$$

$$\omega = \frac{\theta}{t} = \frac{2.5\pi}{1} = 2.5\pi \text{ radians/min}$$

$$\begin{aligned} 1 \text{ revolution} &= 2\pi \text{ radians} \\ 1.25 \text{ revs} &= 1.25(2\pi) \\ &= 2.5\pi \text{ radians} = \theta \end{aligned}$$

Area of a Sector of a Circle

For a circle of radius r , the area A of a sector of the circle with central angle of θ is given by

$$A = \frac{1}{2} r^2 \theta$$

units in $\text{in}^2, \text{ft}^2, \text{cm}^2, \text{etc}$

, where θ is measured in RADIANS

Ex 8: A pizza is about to be split up and you want to be sure you get a lot of it. The diameter is 16 inches and you want a piece with a central angle of 170° . Find the area of the piece you plan to get.

$$170^\circ \frac{\pi}{180^\circ} = \frac{17\pi}{18} \text{ radians}$$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (8)^2 \frac{17\pi}{18} = (32) \frac{17\pi}{18}$$

$$= \frac{272\pi}{9} \text{ in}^2$$

Additional Practice

- 1) Convert from DMS to Decimal Degrees: $57^{\circ}32'45''$

$$= 57.5458\bar{3}^{\circ}$$

$$57 + \frac{32}{60} + \frac{45}{3600}$$

- 2) Convert from Decimal Degrees to DMS: 97.125°

$$= 97^{\circ}7'30''$$

- 3) A carousel with a 50 foot diameter makes 4 revolutions per minute. Find the angular speed of the carousel in radians per minute, and the linear speed of the platform rim.

$$2\pi = 1 \text{ revolution}$$

$$4 \text{ revs} = 4(2\pi) = 8\pi \text{ radians} = \theta$$

$$\text{angular: } \omega = \frac{\theta}{t} = \frac{8\pi}{1} = 8\pi \text{ radians/min}$$

$$\begin{aligned} \text{Linear: } v &= \frac{s}{t} = \frac{r\theta}{t} \\ &= \frac{8\pi(25)}{1} = 200\pi \text{ ft/min} \end{aligned}$$